**Homework 4**

**P15.1.8** (a) Derive the transfer function of the circuit

of Figure P15.1.8; (b) determine *L* and

*C* so as to have a third-order, highpass, normalized Butterworth response, with 1 Ω

resistors; (c) scale the parameters so as to

have resistance values of 2 kΩ and a cutoff frequency .

**Solution:** (a) The transfer function *VO*(*jω*)/*VSRC*(*jω*) will be

determined from TEC looking into terminals ‘ab’.  ; *ZTh* = (*sL*)||(1 + 1/*sC*) = . It follows that:



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=

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(b) The third-order Butterworth response is: (*s* + 1)(*s*2 + *s* +1) = *s*3 + 2*s*2 + 2*s* +1. Comparing coefficients with the denominator, *C* = 1 F so as to have the same coefficients of the *s* term. To have the same coefficients of the *s*2 term, 2 = 2*L* + 1, which gives *L* = 0.5 H. The coefficient of the *s*3 term will be 1.

(c) To have resistance values of 2 kΩ, *km* = 2×103; to have , *kf* = 2*π*×104; hence,  mH, and  7.96 nF.

**P15.2.9** Design a broadband bandpass filter having 3-dB cutoff frequencies of 100 Hz and 10 kHz and a passband gain of 2 using 0.2 μF capacitors. The filter is to have very high input impedance.

**Solution:** For the lowpass filter, the upper 3-dB cutoff frequency is , Ω. For a gain of 2, *Rrl* = 40 Ω. For the highpass filter, the lower 3-dB cutoff frequency is ,  Ω. The high input impedance is provided by a unity gain amplifier. The circuit is s as shown.

**P15.3.3** (a) Derive the transfer

function of the filter

shown in Figure

P15.3.3.

(b) Determine the maximum gain; (c) determine the bandwidth.

**Solution:** (a) the circuit is effectively as shown, with C = 1 nF

*R* = 10 kΩ, *CR* = 10-5 s. For the

amplifier, , where , and , 



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(b) Maximum gain is when *ω* = 104 rad/s, and is .

(c) the bandwidth is the coefficient of *s*, and is 2×104 = 20 krad/s.

**P15.3.7** It is required to design a third-order Butterworth highpass filter using a first-order highpass filter cascaded with a second-order highpass, noninverting filter of the type shown in Figure 15.4.1, reproduced in Figure P15.3.7 using a unity-gain amplifier. The filter should have a gain of 20 dB and a 3-dB cutoff frequency of 10 krad/s, using 0.1 µF capacitors. Determine the required values of resistances.

**Solution:** The third-order Butterworth polynomial is . In the first-order section, *Rr* = 1 kΩ, *Rf* = 10 kΩ to give a gain of 10. For the second-order section, the design equation is (15.4.2):

 , with 

*Q* = 1 so that *R*2 = 2 Ω and *R*1 = 0.5 Ω, *kf* = 104; *km*=. This makes *R*1 =500 Ω and *R*2 = 2 kΩ. The circuit will be as shown.

